CURVATURE OF THE TRANSVERSE ARCH GOVERNS STIFFNESS OF THE HUMAN FOOT

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Summary Humans are unique among primates in having arched feet that provide a stiff propulsive lever for locomotion. Using mathematical and physical models of the foot as a curved elastic shell, we show that the transverse curvature is the primary determinant of foot stiffness. The stiffness of shallow thin shells has two asymptotic regimes, one that resembles a soft thin plate, and the other that shows a power-law dependence of stiffness with an exponent of $3/2$. Curvature induced coupling between bending and in-plane stretching underlies the power-law dependence. Analysis of discrete realizations of a shell also show a similar transition, but with an exponent of 2. We present implications of our work to understanding the mechanical origins of stiff human feet, and to the evolution of human feet through an analysis of extant and fossil feet.

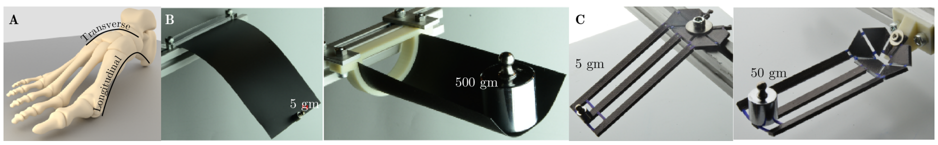

Figure 1: Observations on the effect of curvature on stiffness. **A**. The human foot has two distinct arches in the longitudinal and transverse directions. **B**. Common experience and simple experiments show that curvature in the transverse direction significantly increases the longitudinal bending stiffness of thin continuum shells. **C**. Discrete realizations of a shell, namely, rigid bars connected by soft elastic "ligaments" also shows a similar large increase in stiffness upon increasing curvature in the transverse direction.

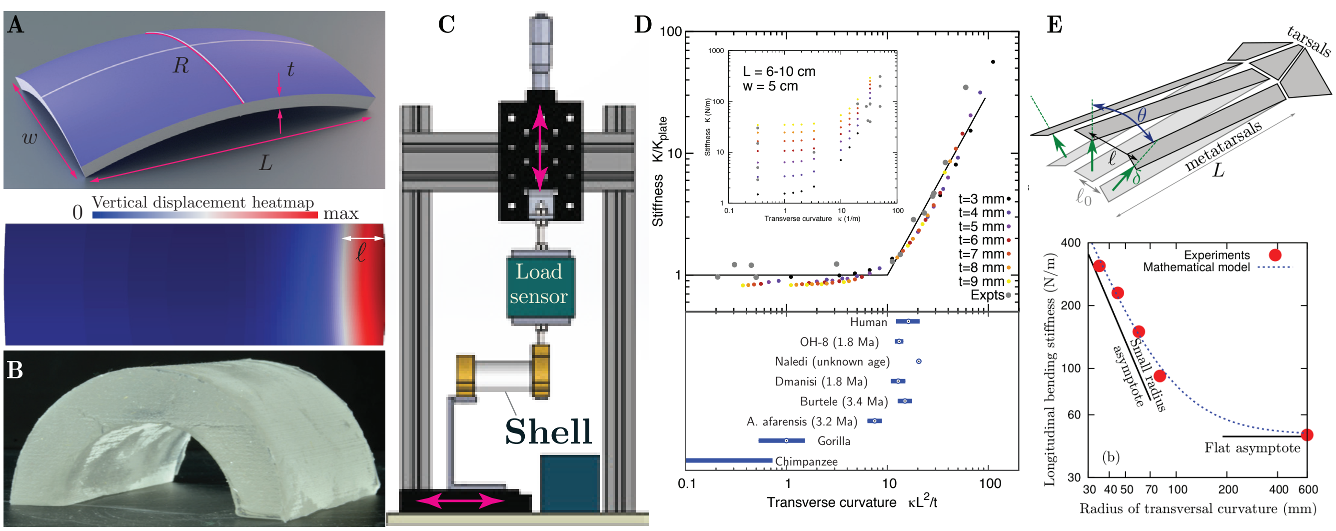

Figure 2: Mathematical and physical analyses of shells, and their implications to human evolution. **A**. Numerical model of curved elastic shells. **B,C**. Experimental fabrication and measurement of stiffness of transversally curved shells. **D**. The inset shows dimensional data, that collapse according to the asymptotic predictions. Comparing the dimensionless curvature of various biological feet, relative to the crossover point of the asymptotes, predicts that a human-like stiff foot might have emerged at least 3.4 million years ago. **E**. Ongoing work to address the mechanics of discrete mimics of the foot, with stiff "bones" and softer interconnecting "ligaments".

Just as a drooping dollar bill stiffens upon curling it in the transverse direction (figure 1B), we hypothesize that the transverse arch of the foot plays a central role in longitudinal bending stiffness. We observe a similar increase in stiffness for physical mimics of the foot, which resemble a discrete realization of the dollar bill (figure 1C).

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MODELS OF THE FOOT

Mathematical models: We modeled the foot computationally (using COMSOL) and experimentally (static loading experiments) as a continuum elastic shell of length L, width w, and thickness t. The shell has radii of curvature R_L and R_T in the longitudinal and transverse directions respectively (figure 2A,B). To isolate the dependence of the stiffness K of the shell on the geometrical parameters L, w, t, R_L, and R_T, but eliminate material nonlinearities, the shell is assumed to be made of a uniform linear elastic solid with Young's modulus E and Poisson ratio ν.

For flat thin plates, it is well known that stretching can be neglected and is decoupled from bending if the deflection of the end is much smaller than the thickness [Landau and Lifshitz(1959)]. For curved shells, however, stretching with bending could be a dominant effect even for small out-of-plane deflections. For a radial displacement by an amount w, the strain scales as $\sim w/R$, where R is the typical radius of curvature. The elastic stretching energy therefore scales as $\mathcal{E}_{\rm s} \sim EtA(w/R)^2$. When deformations are uniform, pure bending energy scales as $\mathcal{E}_{\rm b} \sim Et^3A(w/R^2)^2$, always resulting in a large stretching-to-bending ratio, $\mathrm{r} = \mathcal{E}_{\rm s}/\mathcal{E}_{\rm b} \sim (R/t)^2 \gg 1$.

When the deformation is caused by an external normal force applied to the shell and the characteristic length scale of the range of this deformation is ℓ, bending energy will scale as $\mathcal{E}_{\rm b} \sim Et^3 lW(w/\ell^2)^2$, where W is the projected width of the shell. Numerical experiments indeed show the localization of the deformation for curved shells (figure 2A). Hence, by balancing the stretching and bending energies (equivalent to minimizing the total elastic energy), we derive a scaling law for the deformation depth, namely $\ell \sim \sqrt{Rt}$. Therefore, for some function f depending on the precise loading of the shell, dimensional analysis implies $\frac{K}{Bw/L^3} = f(R_Tt/L^2, R_Lt/L^2)$, where the bending rigidity of the cross section of the shell is $B = Et^3/(12(1-\nu^2))$.

Physical models: Experimental realization of this shell was made using the soft elastomer polydimethylsiloxane (PDMS) (figure 2B). The shell was clamped at one edge, and an external force was exerted on the other, to mimic the loading experienced by the foot during toe-off (figure 2C). The ratio of the applied force to the relative displacement between the edges of the shell in the limit of small displacements gives stiffness K.

RESULTS

The inset of figure 2D shows the calculated and measured stiffness K of the foot for shells with different geometries. Using the dimensionless forms of stiffness and curvature cause these data to collapse, in agreement with the asymptotic predictions. There was no such effect on longitudinal bending stiffness because of longitudinal curvature. We also plot the curvature of human, gorilla, chimpanzee and several fossil feet [Harcourt-Smith and Aiello(2004), Pontzer et al.(2010)], shown in the lower half of figure 2D. Among living primates, humans are the only feet that belong to the 3/2 power-law regime, i.e. correctly predicted to exhibit a stiff shell-like behavior [Ker et al.(1987)]. On the other hand, the gorilla and chimpanzee are flat, and correctly predicted to be soft like a thin plate [Bennett et al.(1989)]. The fossil feet show a clear increase in curvature at least 1 million years (Ma) before the emergence of the genus *Homo*. The ~ 3.4 Ma foot (Burtele, Ethiopia), which was previously thought to resemble the gorilla, has a human-like arch even if we assume its proportions to be that of a gorilla.

CONCLUSION

The curvature of the transverse arch in humans is expected to contribute to a 200% increase in stiffness, all else held the same. Human feet are approximately 220% stiffer than chimpanzee feet [Bennett et al.(1989)], after adjusting for size differences and assuming equal Young's modulus (using our dimensionless stiffness). In contrast, the elastic tissue that constitute the longitudinal arch account for less than 30% of the human foot stiffness [Ker et al.(1987)].

Measurements of stiffness as a function of curvature using the discrete foot shown in figure 1C are in agreement with theoretical predictions (with two fitting parameters). However, the power-law now exhibits an exponent of 2, different from the continuum model. We are continuing to investigate the origins of this difference in the exponent.

Our results aid in the study of fossil feet, to infer function from form. There are also implications for flatfoot disorders, and for the the design of lightweight, yet stiff, robotic or prosthetic feet. Finally, we propose that control of the transverse curvature is an attractive method to modulate foot stiffness because of the strong dependence of stiffness on curvature.

References

[Bennett et al.(1989)] M. B. Bennett, R. F. Ker, and R. M. Alexander. *J. Zool*, 217:469–475, 1989.

[Harcourt-Smith and Aiello(2004)] W. E. H. Harcourt-Smith and L. C. Aiello. *J. Anat*, 204(5):403–416, 2004.

[Ker et al.(1987)] R. F. Ker, M. B. Bennett, S. R. Bibby, R. C. Kester, and R. M. Alexander. *Nature*, 325(7000):147–9, 1987.

[Landau and Lifshitz(1959)] L. D. Landau and E. M. Lifshitz. Pergamon Press, 1959.

[Pontzer et al.(2010)] H. Pontzer, C. Rolian, G. P. Rightmire, T. Jashashvili, M. S. Ponce de León, D. Lordkipanidze, and C. P. E. Zollikofer. *J. Hum Evol*, 58(6):492–504, 2010.