

# BUCKLING AND COLLAPSE OF THE BICYCLE WHEEL

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**Summary** A bicycle wheel is a prestressed structure; the tensioned spokes hold the rim under compression. It can buckle due to the elastic energy stored during construction or from external loads. This buckling mode, known as a “taco” or “potato chip” due to its saddle-like shape, can result from either of two related failure mechanisms: (1) elastic buckling under uniform spoke tension, and (2) dynamic collapse when loaded through the hub. Dynamic failure initiates from local buckling of the rim near the road contact. We derive an analytical expression for the buckling tension for an unloaded wheel and simulate wheel collapse using non-linear finite-element calculations. We present a lower bound for the load at which spokes will buckle. We find that increasing spoke tension increases the failure load, unless the spoke tension is close to the buckling tension, in which case the wheel will collapse under even a small disturbance.

## ANALYSIS AND DISCUSSION

The rim of a conventional bicycle wheel is connected to the hub by a set of slender spokes. Each spoke exerts a restoring force on the rim proportional to the deflection of the rim at that point. If the spokes are closely spaced, the rim can be modeled as a curved beam on a continuous elastic foundation [1]. The foundation stiffness in the radial and lateral directions are

$$k_{rr} = \frac{n_s E_s A_s}{2\pi R L_s} \cos^2 \alpha, \quad k_{uu} = \frac{n_s E_s A_s}{2\pi R L_s} \sin^2 \alpha \quad (1)$$

where  $n_s$  is the number of spokes,  $E_s$  and  $A_s$  are the Young’s modulus and cross-sectional area of a spoke,  $R$  is the rim radius,  $L_s$  is the length of a single spoke, and  $\alpha$  is the inclination angle of a spoke out of the center plane of the wheel.

### Elastic buckling under uniform spoke tension

We first derive an analytical solution for the critical tension based on the principle of stationary potential energy. The strain energy is decomposed into components for rim bending, torsion, spoke deformation, and a non-linear contribution from the compressive force in the rim,  $U = U_{bend} + U_{tors} + U_{spokes} + U_{nonlinear}$  [2]. Decomposing the deformation into a series of sinusoidal modes and setting the second variation  $\delta^2 U = 0$  leads to an expression for the critical buckling tension.

$$T_{max} = \frac{2\pi EI}{n_s R^2} \left( \frac{\mu(n^2 - 1)^2}{1 + \mu n^2} + \frac{k_{uu} R^4}{EI n^2} \right) \quad (2)$$

where  $EI$  is the rim bending stiffness,  $\mu$  is the ratio of the torsion to bending stiffness  $GJ/EI$ , and  $n$  is the mode number.

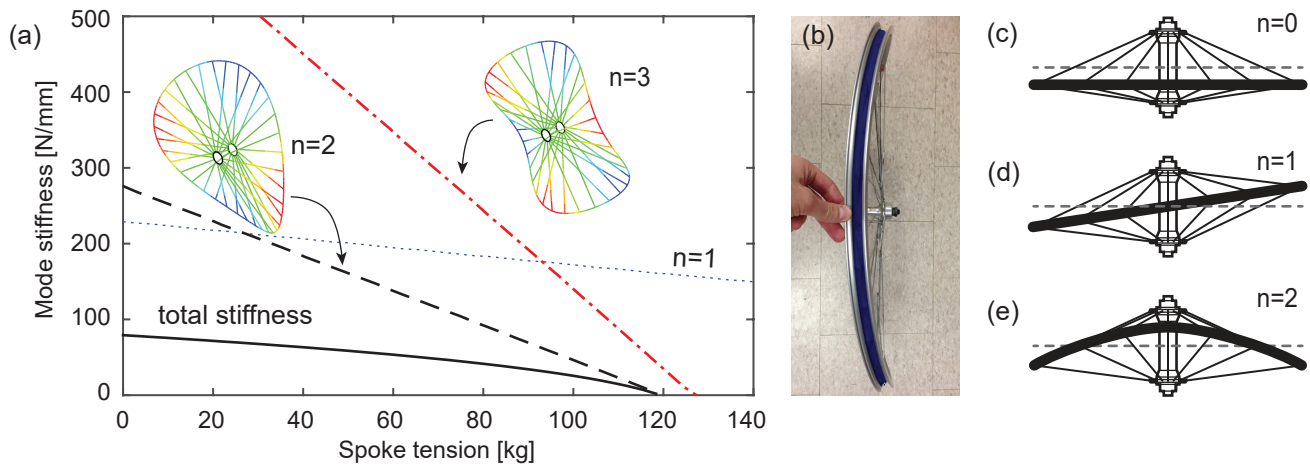


Figure 1: (a) Mode stiffness for the first three rim modes and the overall wheel stiffness. The wheel buckles when the stiffness of mode  $n=2$  goes to zero. (b) Buckled wheel viewed from above. (c-d) Sinusoidal deformation modes.

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### Dynamic collapse of a loaded bicycle wheel

When a bicycle wheel supports a vertical load, the load is supported by shortening of a few spokes underneath the hub [3]. If the load is great enough to cause spokes in this zone to lose tension entirely leaving the rim unsupported laterally, the wheel will buckle locally which leads to catastrophic failure. Dynamic finite-element simulations of wheel collapse reveal a failure process very similar to that which was qualitatively described by Brandt [4]. The failure process is modeled in ABAQUS Explicit using beam elements for the rim and spokes (Figure 2).

The load-displacement curve for a loaded wheel exhibits non-linearity due to buckling of individual spokes underneath the hub. We model this behavior by including a foundation yield stress,  $f_y = n_s T / 2\pi R$  in the continuum elastic foundation model. The yield stress represents the average line load at which the spoke loses tension. We implement this model using a custom non-linear finite-element code (Figure 2 (c)). A lower bound for the onset of spoke buckling is given by

$$P_{crit} = 2f_y \sqrt[4]{4EI/k_{rr}}. \quad (3)$$

The collapse load depends on the rim and spoke properties, as well as the spoke tension. Increasing spoke tension generally increases the buckling load because the spoke system can bear a higher load before detensioning (note the non-linear behavior in Figure 2 (c)). However, if the spoke tension is close to the critical buckling tension predicted by Equation 2, even a small disturbance will lead to failure.

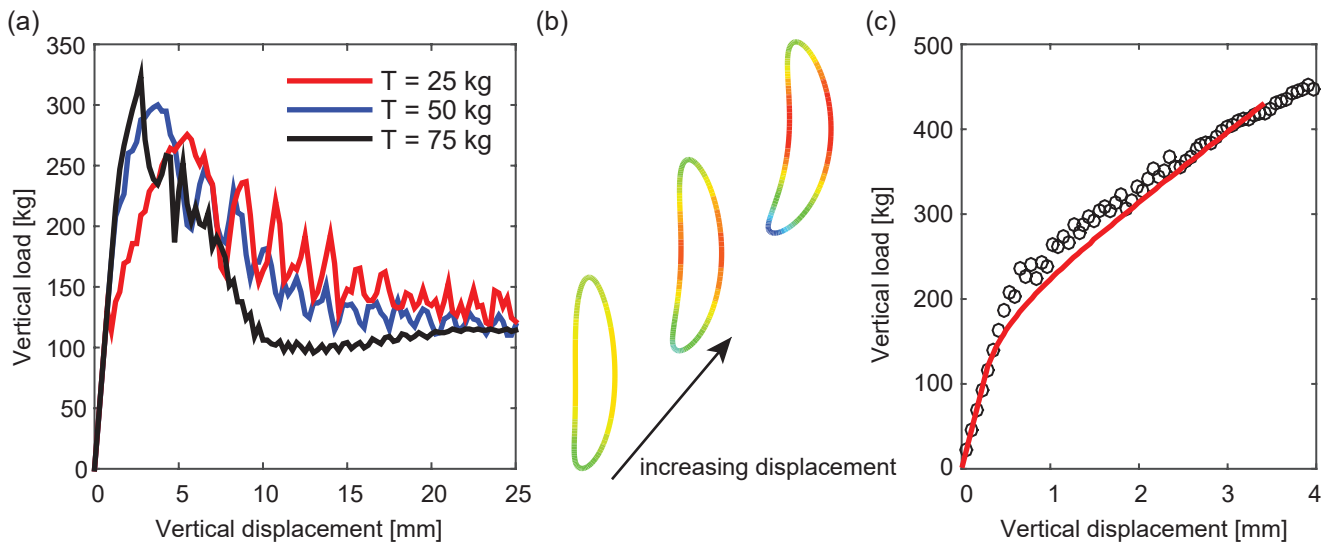


Figure 2: (a) Load-displacement curve for wheels with different spoke tensions. (b) Snapshots from a simulation of wheel collapse. Spokes are hidden for clarity. (c) Comparison between direct numerical solution with ABAQUS Explicit (circles) and continuum beam on elastic-plastic foundation model (red line).

### CONCLUSIONS

The strength of a bicycle wheel is optimized through cooperativity between the rim and the spoke system. The spokes must be tight enough to resist detensioning when the wheel is loaded and the rim must be stiff enough to distribute the load over several spokes. Tightening the spokes beyond the critical tension described by Equation 2 will cause the wheel to buckle without any external load. We described the mechanism of wheel collapse through numerical simulations and derived a lower bound for the critical load for spoke buckling in Equation 3. These results can be used by designers and wheelbuilders to optimize wheel strength for different riders and conditions.

### References

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