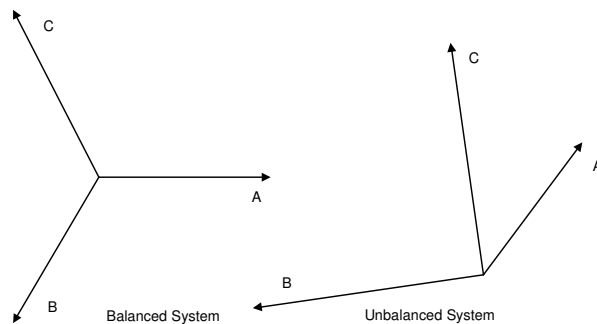


PHASOR DIAGRAMS II

Fault Analysis

Ron Alexander – Bonneville Power Administration

For any technician or engineer to understand the characteristics of a power system, the use of phasors and polarity are essential. They aid in the understanding and analysis of how the power system is connected and operates both during normal (balanced) conditions, as well as fault (unbalanced) conditions. Thus, as J. Lewis Blackburn of Westinghouse stated, “a sound theoretical and practical knowledge of phasors and polarity is a fundamental and valuable resource.”



With the proper identification of circuits and assumed direction established in a circuit diagram (with the use of polarity), the corresponding phasor diagram can be drawn from either calculated or test data. Fortunately, most relays today along with digital fault recorders supply us with recorded quantities as seen during fault conditions. This in turn allows us to create a phasor diagram in which we can visualize how the power system was affected during a fault condition.

FAULTS

Faults are unavoidable in the operation of a power system. Faults are caused by:

- Lightning
- Insulator failure
- Equipment failure
- Trees
- Accidents
- Vandalism such as gunshots
- Fires
- Foreign material

Faults are essentially short circuits on the power system and can occur between phases and ground in virtually any combination:

- One phase to ground
- Two phases to ground
- Three phase to ground
- Phase to phase

As previously instructed by Cliff Harris of Idaho Power Company: Faults come uninvited and seldom leave voluntarily. Faults cause voltage to collapse and current to increase. Fault voltage and current magnitude depend on several factors, including source strength, location of fault, type of fault, system conditions, etc. Faults must be isolated from the power system quickly in order to minimize damage to equipment, the environment, or the power system in general, as well as to eliminate the hazard to people. Fault angle is the angle of the fault current relative to its respective voltage. The angle of the fault current (PF) is determined for phase faults by the nature of the source and connected circuits up to the fault location. For ground faults, you also must consider the type of system grounding.

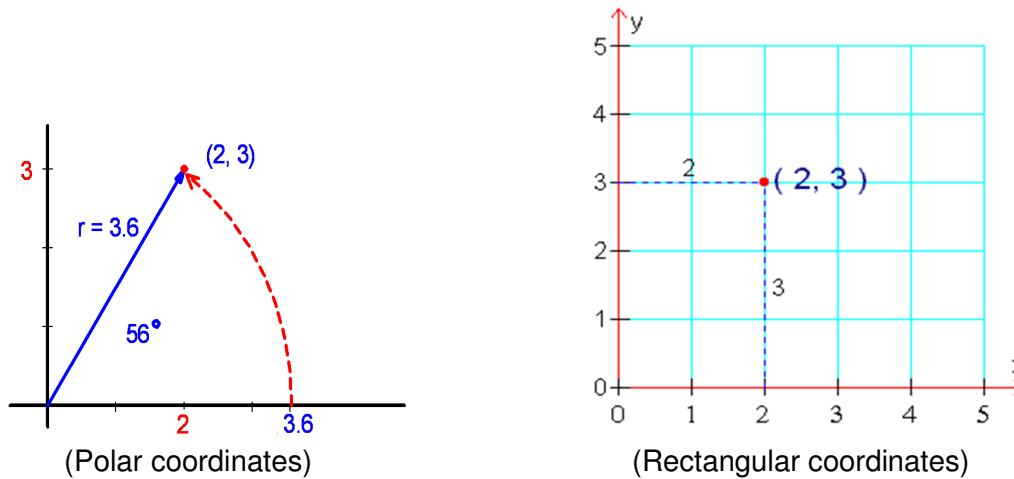
Typical fault angles on open wire transmission lines are:

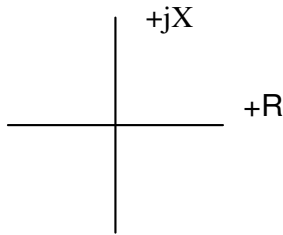
- 7.2-23kV = 20-45° lag
- 23-69kV = 45-75° lag
- 69-230kV = 60-80° lag
- 230kV and up = 75-85° lag

For faults on transmission lines, this angle is a function of the characteristics of the transmission line. High voltage transmission lines generally utilize large conductors, which characteristically have high inductance and low resistance. Thus the fault angle (current lagging voltage) will be high, usually in the range of 70-85°. Lower voltage transmission lines usually employ smaller conductor, with higher resistance than larger conductors. Typical line fault angles are in the 40-70° range. Fault resistance, especially with grounded faults, needs to also be taken into consideration too as caused by tower footings, tree limbs, ground, arc length through air, or other factors can also influence the fault angle.

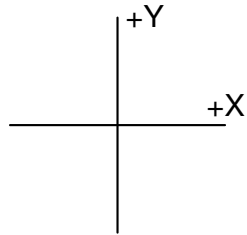
Key points on making phasor diagrams from J. Lewis Blackburn’s Protective Relaying – Principles and Application, Chapter 3:

Common pictorial form for representing electrical and magnetic phasor quantities uses the Cartesian coordinates with x (the abscissa) as the axis of the real quantities and y (the ordinate) as the axis of imaginary quantities. (see figures below)

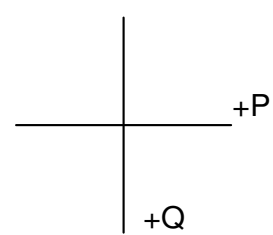




Impedance



Current & Voltage



Power

In normal practice, the phasor represents the rms maximum value of the positive half cycle of the sinusoid unless otherwise specifically stated.

Phasors, unless otherwise specified, are used only within the context of steady state alternating linear systems.

The term “phasor” can also be applied to impedance, and related complex quantities that are not time dependent.

While represented as phasors, the impedance and power “phasors” do not rotate at system frequency.

The international standard is that phasors always rotate in the counterclockwise direction. However, as a convenience, on the diagrams the phasor is always shown “fixed” for the given condition.

Phasor diagrams require a circuit diagram. The phasor diagram... has an indeterminate or vague meaning unless it is accompanied by a circuit diagram.

The assumed directions and polarities are not critical, as the phasor diagram will confirm if the assumptions were correct, and provide the correct magnitudes and phase relations.

These two complementary diagrams (circuit and phasor) are preferably kept separate to avoid confusion and errors in interpretation.

Nomenclature for Current and Voltage: ‘Unfortunately there is no standard nomenclature for current and voltage, so confusion can exist among various authors and publications. The nomenclature in [Blackburn’s] book has proven to be flexible and practical over many years of use, and it is compatible with power system equipment polarities.

Current and Flux. The direction [of current or flux] is that assumed to be the flow during the positive half cycle of the sine wave.

The single subscript... is a convenience to designate currents in various parts of a circuit and has no directional indication, so an arrow for the direction must be associated with these.

Arrows are not required with [double subscripts] but are often used for added clarity and convenience.

It is very important to appreciate that in these circuit designations, the arrows do not indicate phasors. They are only assumed directional and location indicators.

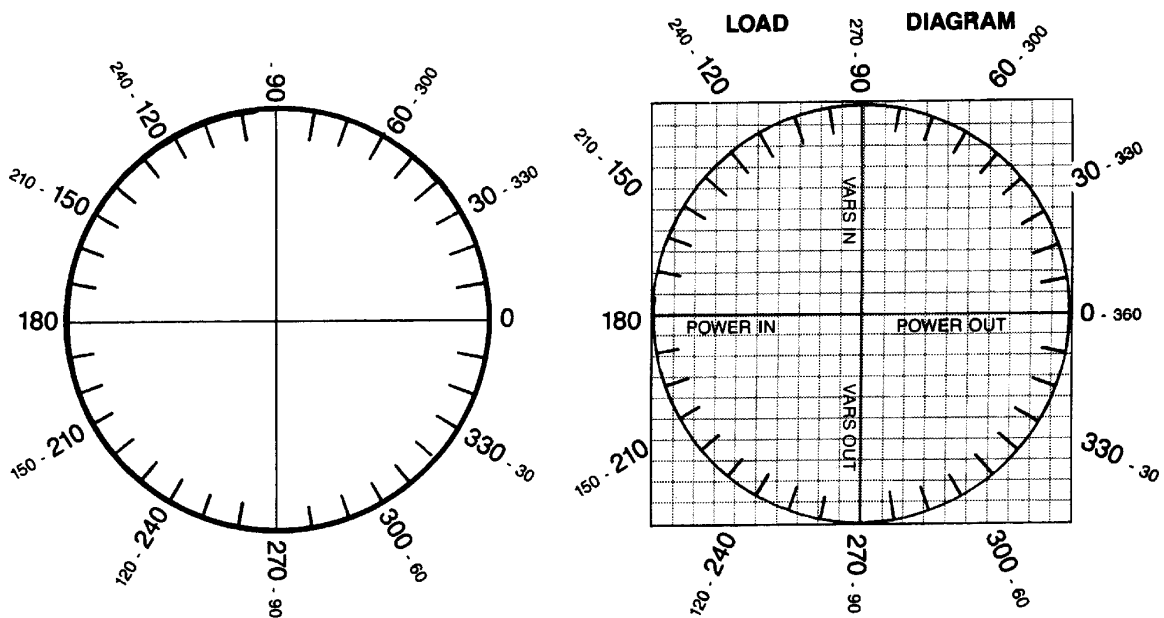
Voltage: Voltages can be either drops or rises. Much confusion can result by not clearly indicating which is intended or by mixing the two practices in circuit diagrams. This can be avoided by standardizing on one and only one practice. As voltage drops are far more common throughout the power system, all voltages are shown and always considered to be drops from a higher voltage to a lower voltage. This convention is independent of whether the letter V or E is used for the voltage.

Voltages (always drops) are indicated by either (1) a letter designation with double subscripts, or (2) a small plus (+) indicator shown at the point assumed to be at a relatively high potential.

It may be helpful to consider current as a “through” quantity and voltage as an “across” quantity.

Phasor and Phase Rotation “Phasor” and “phase rotation” are two entirely different terms, although they almost look alike. AC phasors are always rotating counterclockwise at the system frequency.

In contrast, phase rotation or phase sequence refers to the order in which the phasors occur as they rotate counterclockwise. The standard sequence today is a, b, c; A, B, C; 1, 2, 3; or in some areas R, S, T.

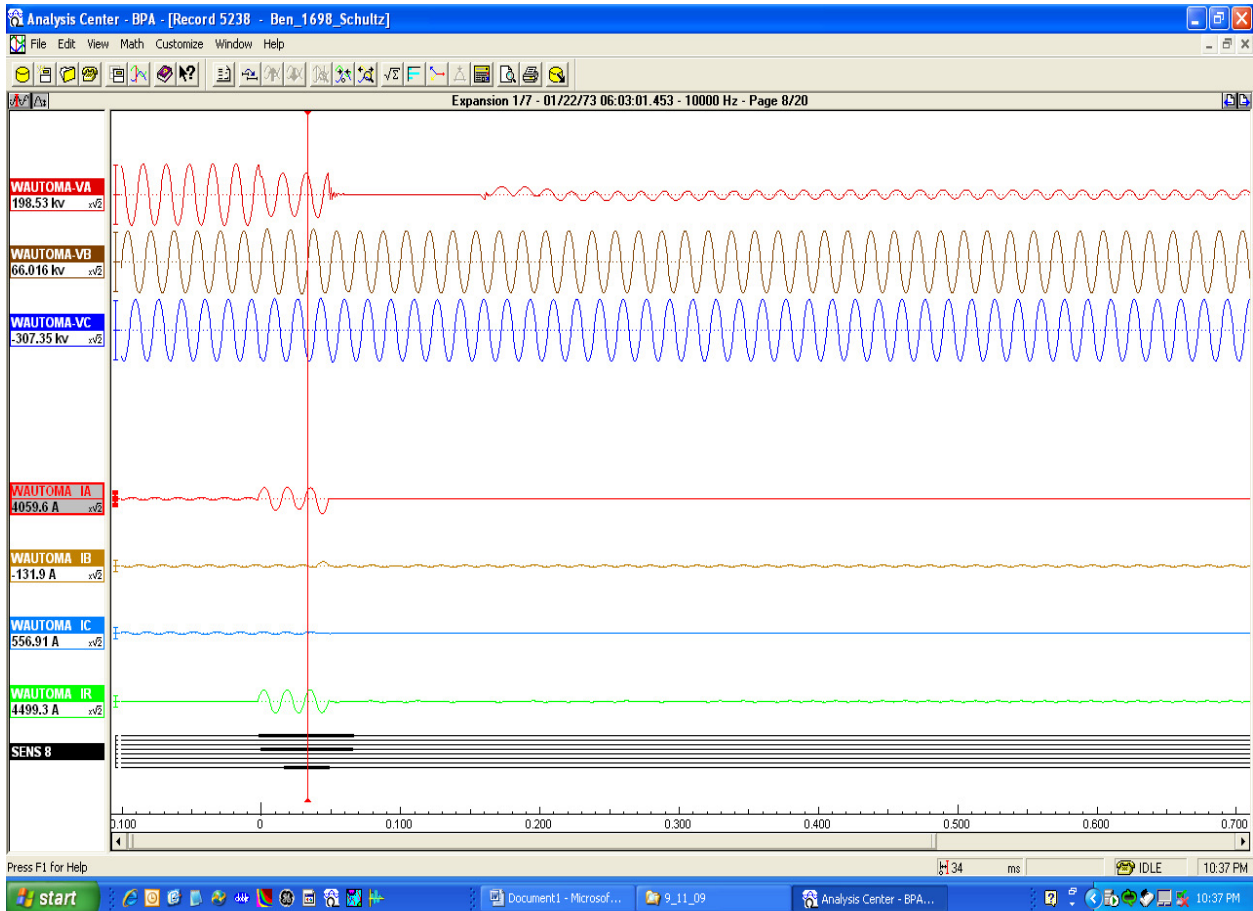


CLASSROOM LESSONS:

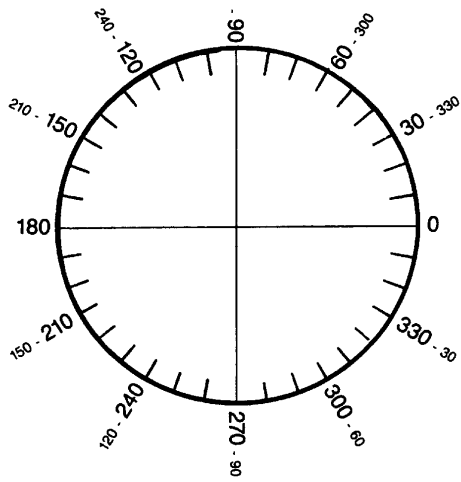
Let's try applying what we have learned by drawing in the classroom a phasor diagram for each of the given fault types. We can make a diagram for pre-fault, fault, and post-fault. This is an open discussion!

SINGLE PHASE FAULTS

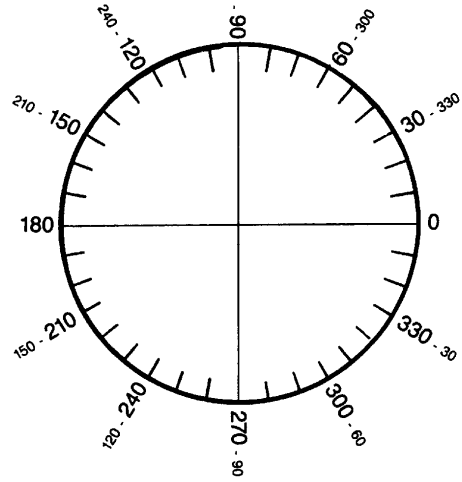
Single phase faults are the most common. When viewing the faulted line, the phase voltage collapses, and its current increases while also lagging the voltage by some value.



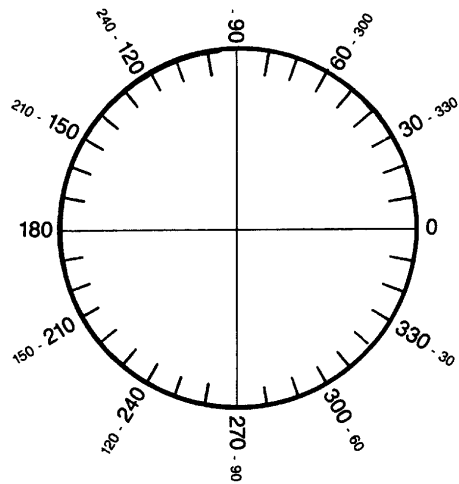
Pre-Fault



Fault



Post-Fault



3 PHASE FAULTS

A 3 phase fault reduces all three voltages and causes a large increase and usually highly lagging fault current symmetrically in all three phases. The angle of the lag is determined by the system.



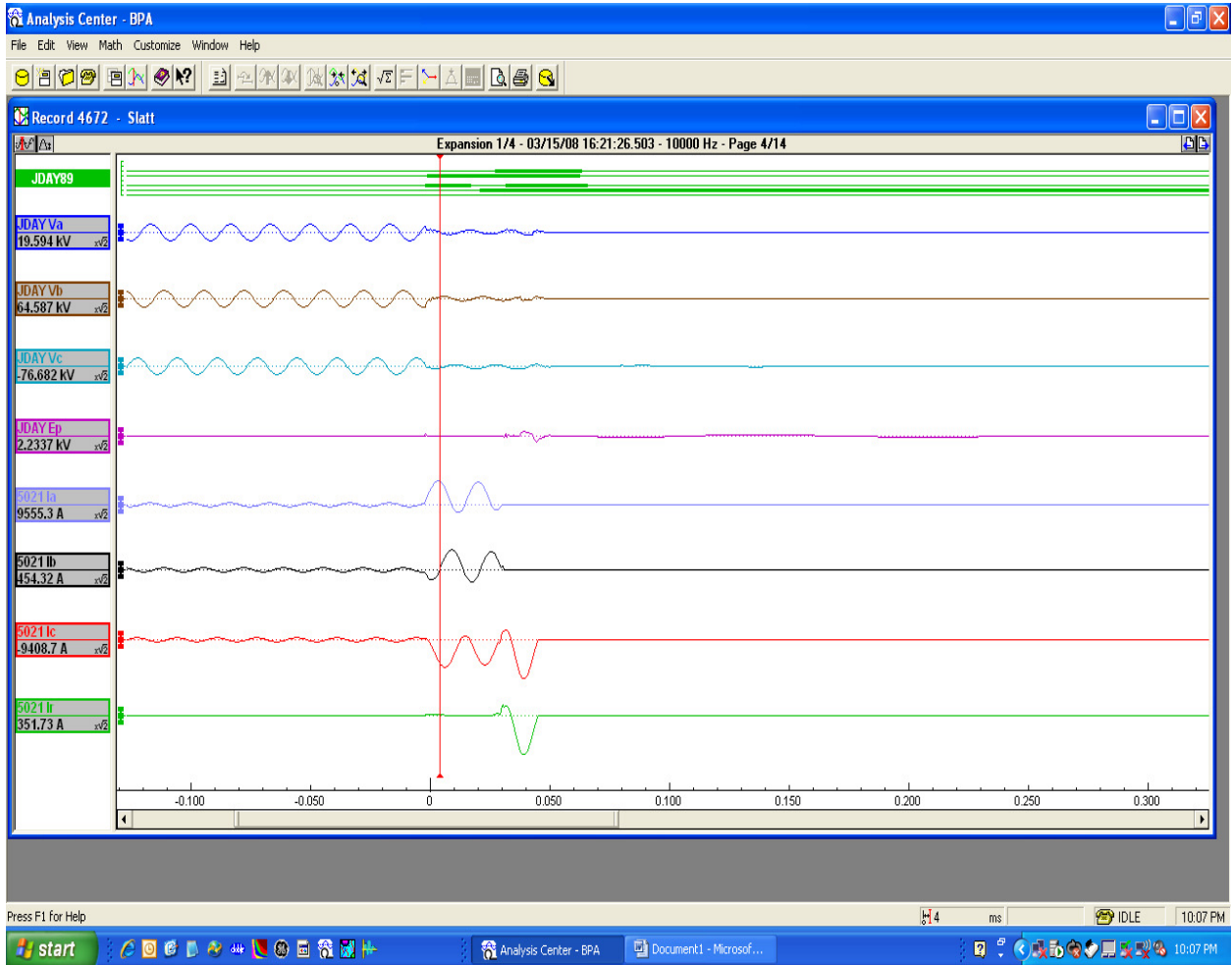
Now, let's reverse our thinking and write in on the above graph what you think a three-phase fault would look like.

Use these values:

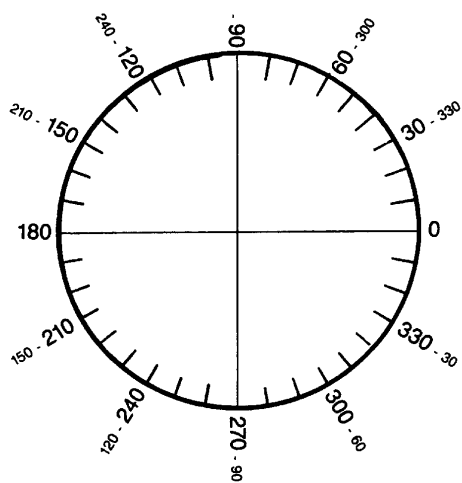
Voltages shrink from 115kV to 54kV line – ground. What phase angles might they be at???

Current increases from of about 200A to what???. And what phase angles might they be at???

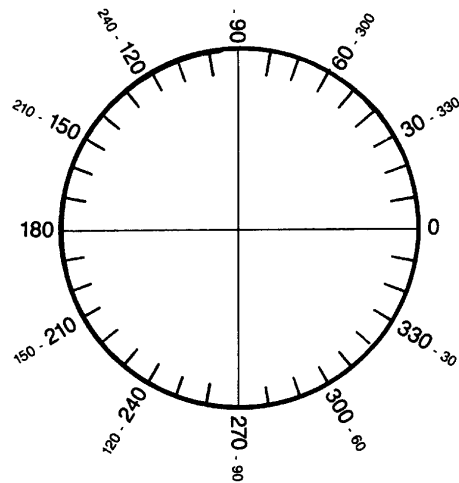
This is a real 3 phase fault. What do you see? Is it different than what you predicted? Why?



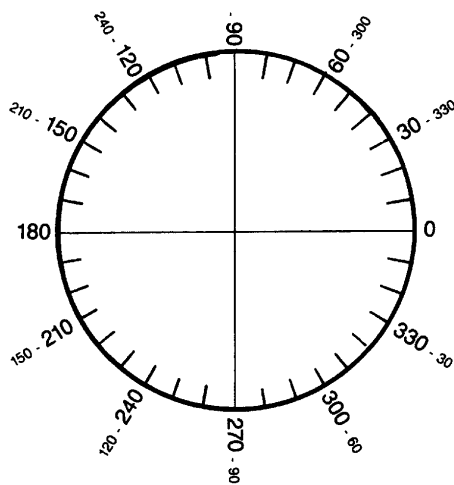
Pre-Fault



Fault

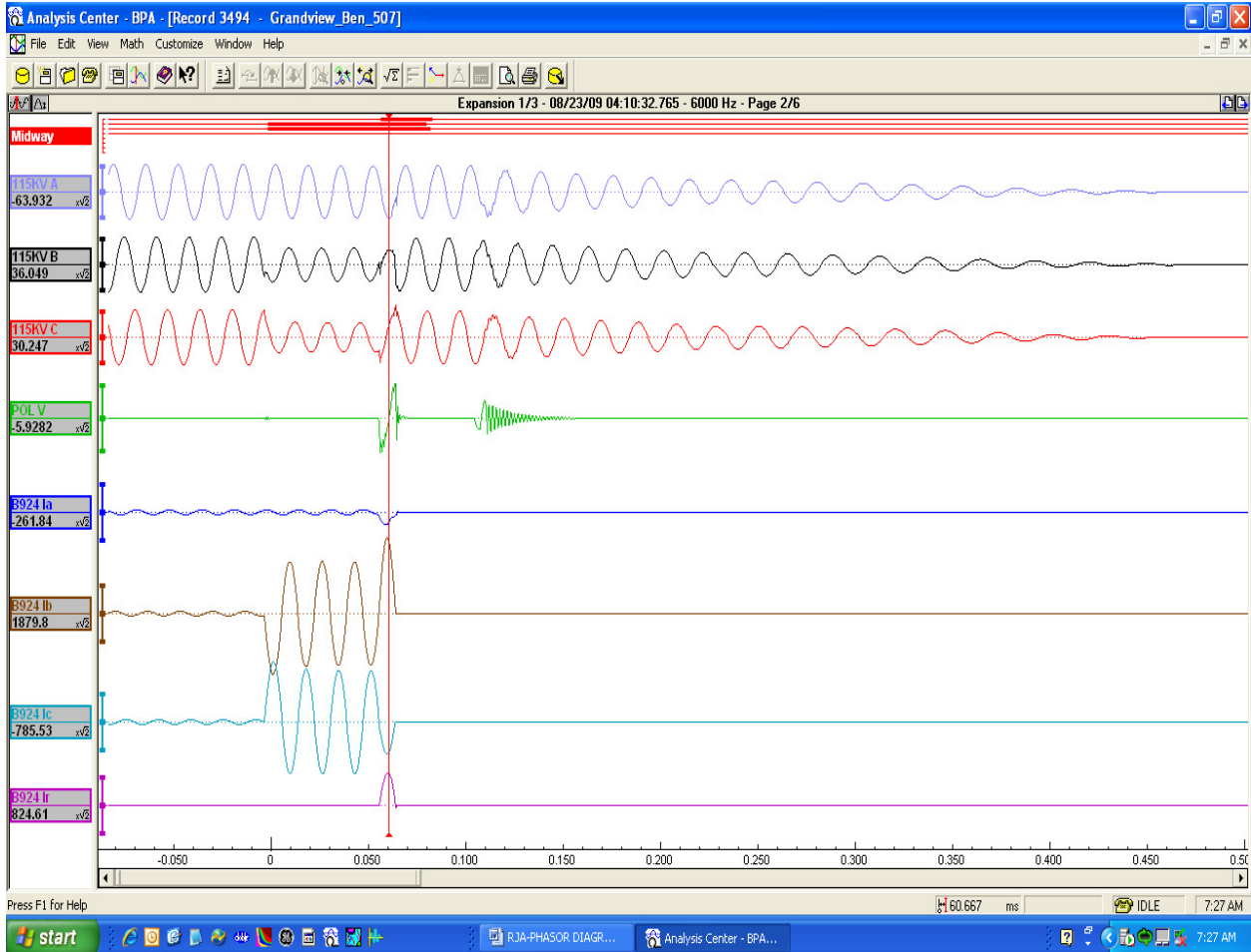


Post-Fault

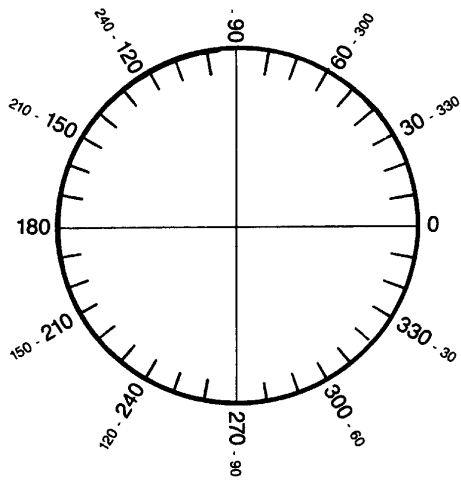


PHASE TO PHASE FAULTS

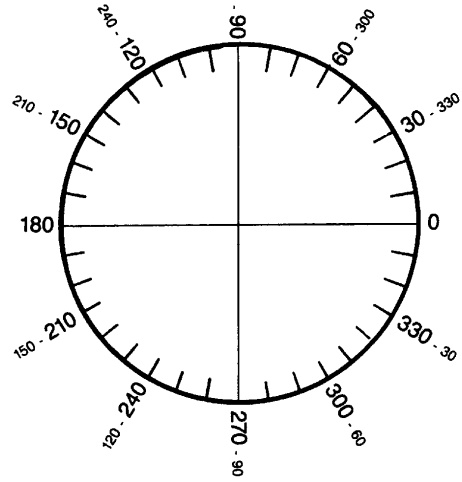
Phase to phase faults are sometimes the most difficult to understand. The faulted phase voltages (i.e.: b and c phases) collapse from their normal position and the phase angles can swing toward each other. During that time, current magnitudes increase as one current will lag the faulted phase to phase voltage value (V_{bc}) while the other phase current will be 180° out.



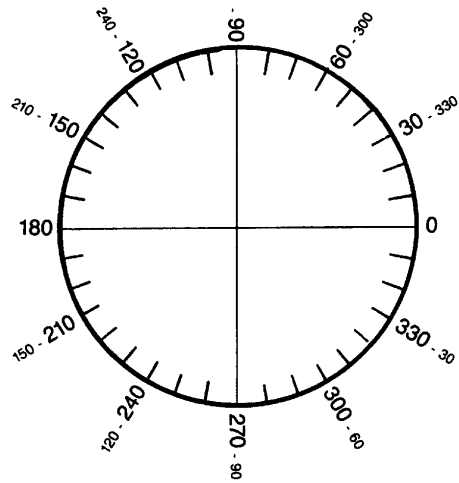
Pre-Fault



Fault



Post-Fault



SUMMARY NOTES FOR PHASORS AND IN-SERVICE READINGS

INTRODUCTION TO PHASORS

Phasor Notation

A standard naming convention for phasors (labeling and short hand notation) is as follows:

(1) Voltage phasor notation subscript labels signify a head minus tail instantaneous voltage difference (potential drop between point 1 and point 2). e.g., point A and point B.

(2) Current phasor notation subscript labels signify a tail towards head instantaneous current direction (current flowing from point 1 to point 2). e.g., point A to point B.

The following are examples of phasor labeling and its corresponding shorthand notation.

$N \xrightarrow{V} A$ Notation is $V_A - V_N = V_{AN}$

$O \xrightarrow{V} A$ Notation is $V_A - V_O = V_{AO}$

$B \xrightarrow{V} A$ Notation is $V_{AN} - V_{BN} = V_{AB}$

$A \xrightarrow{I} B$ Notation is current flow from V_{AN} to $V_{BN} = I_{AB}$

Note: V_A and I_A are short hand for V_{AN} and I_{AN} (the wye equivalent voltage and current).

Although the above naming convention is now considered standard, somewhat anyway, a few textbooks show subscript notation reverse. That is, V_{AO} is shown as V_{OA} or V_{AB} is shown as V_{BA} . This reverse notation simply means they go tail to head when naming the voltage vectors (phasors). The "O" subscript is just indicating the opposite end (or origin) of the vector.

Even though there are two basic ways of assigning double subscript notation, which adds confusion, both formats are correct if all your vectors are assigned against the same reference direction. Why are they both valid? This is because AC measurements oriented against the negative half of a sinusoidal wave look identical to those oriented against the positive half. In other words, if you connect your test circuit reference up reverse, and take all your readings reverse, the phasor readings would be the same as if they were all referenced and taken forward. Unfortunately, the confusion doesn't stop here.

Some text books will show notation indicating the direction of voltage rise for generating sources and the direction of voltage drop for loads, but utilize a different character symbol (E or V) for the type of voltage. For example, E_{NA} for a generator source and V_{AN} for a load connected to the generator. "E" being an EMF source and "V" being a voltage drop. You may also find subscripts (such as for V_{AO}) described as $V_{AA'}$ or V_{Aa} .

A few textbooks use \longrightarrow (open arrows) for voltage phasors and $\longrightarrow\triangleright$ (closed arrows) for current phasors for graphical clarity. You might want to do the same with your drawings, because it makes it easier to distinguish between vectors drawn close to or at the same angle. Regardless of how you draw vector arrowheads, correct and consistent labeling is also important. More importantly, select a single convention and stick to it, because it will reduce the chance of making a mistake. It can even help to think of the notation from a point of making voltage and current measurements.

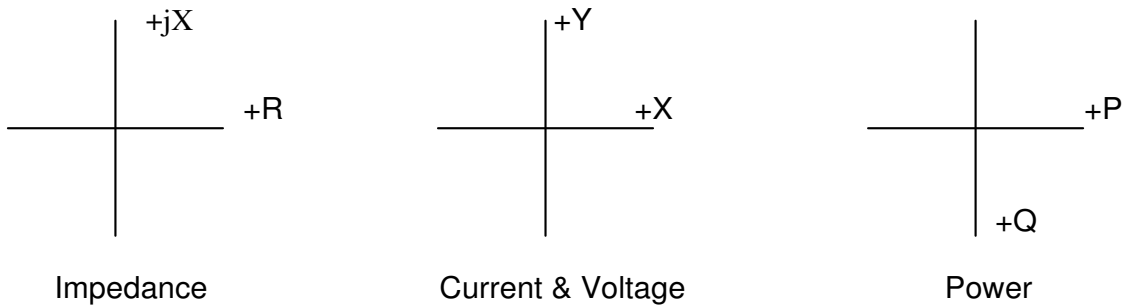
In assigning subscript notation, it may help to think of how you would instrument and measure the circuit with a voltmeter, ammeter, and phase angle meter. Since the inputs of your measurement instruments represent an actual resistive load, they generate the same instantaneous voltage across their terminals as the system being measured. The instantaneous direction across the phase angle meter terminals will align with the instrument polarity markings (if current and voltage leads are connected correctly). Because of this, you can verify your diagram and notation labels by visualizing how you would connect test leads to take “in-service” measurements. For voltage, you would want to visualize this as your red (\pm) “polarity” lead from your test set, or phase angle meter, goes to the first subscript character and black “non-polarity” lead goes to the second character. (This is analogous to a DC voltage measurement). For current, connections to the test instrument would be such that current flowed into the red (\pm) “polarity” lead from the first subscript notation and out the black “non-polarity” lead to the second notation character. (This is also analogous to DC as a current measurement). For current lead connections, you would be inserting a low resistance shunt between the notation subscripts. From a voltage only point of view, the measurement for current ends up looking the same as for voltage, since you have to break open the circuit and insert a burden that develops a voltage in your test instrument. An additional note: *When measuring DC values with a DC meter, red to positive and black to negative would read as a positive voltage (analogous to 0° AC), and a reversed connection would read negative (analogous to 180° AC). For a DC meter, DC indication is also a mathematical difference in potential (red input “minus” black input = measured voltage).*

In summary, the double subscript labeling convention for *voltage* is voltage drop from point A to point B (V_{AB}), and *current* is labeled for direction as flowing from point A to point B (I_{AB}). For example, I_{AN} is current flowing from point A to point N and is said to be flowing from phase A to the Neutral connection. Please note that line current is truly phase-to-neutral current for wye connected systems, and is also the current of an “equivalent wye” in delta connected systems. This is why I_A can be short hand for I_{AN} when talking line current.

Coordinate Grids

Most utilities, such as BPA, also have a convention for drawing phasors on an “x - y” coordinate grid. Except for the positive direction of reactive power, they follow standard academic convention. Even so, if you prefer to explain power with positive reactive power pointing up, as done in academic textbooks, your vector mathematics will take the form of $S = V I^*$ for plotting Volt-Amps. “S” is apparent power, and “*” means to conjugate the current phasor angle before multiplying voltage and current vectors. Conjugate means to reverse the sign of the phasor angle (change to + if -, or to - if +). To eliminate the need for changing the sign of an angle, and making extra calculations, phasor drawings indicate reactive power flow as positive when pointing down.

Using this style of power flow coordinates allow the vector direction of current to describe the direction of real and reactive power flow (“+” = OUT; “-” = IN). Pointing inductive power flow down, is a quick way to show all of the needed information while making it easy to interpret in-service readings (of volts, amps, and phase angle). The quadrant your current falls in also indicates the directions of real and reactive power flow. In practice, layout and labeling on your in-service test sheets will already do this for you.



A quick comment about in-service measurements that is related to plotting phasors on coordinate grids. Try to maintain a single source as your reference for a complete set of in-service readings. This will assure that everything measured is comparable against any other measurement, since they will all have a common reference. Of course, this is not always possible, because of physical distance between measurement points (such as measurements at two different substations). In cases where different references are used, and a phasor comparison is needed between the two sets of measurements, the reference sources will need to be linked by an independent but verifiable quantity. This is typically done by selecting a common power system phase voltage, such as “A” phase, because larger utilities maintain common connections (per phase) across the entire network of 115 kV through 500 kV systems. On lower kV systems, you are not going to find yourself so lucky, so watch what you use as a measurement reference.

Plotting Phasors From In-Service Measurements

Use a similar standard convention when describing “phasor” vectors, because it will keep you from making a directional error if you stick to only one convention. Phasor notation is based on measuring system quantities at a substation and treating each measurement as if it were for a load. The connection point of your voltage is your reference, and all current measurements are taken assuming current flow away from the voltage reference point (i.e., all the lines connected to a substation bus). Because a power system is bi-directional, meaning power flow can be either forward or reverse, system current is also. You must “assume” a standard direction for all measurements, take all measurements as if voltage and current are existing in the assumed direction, and let the readings tell you which way currents are actually flowing. Current “OUT” will be in-phase with voltage and current “IN” will be 180° out-of-phase. Basically, you treat every line connected to a bus as a load, regardless of whether you know it is or isn't.

Here is a case in point:

If in-service readings were taken on both ends of a transmission line, and done correctly by two individuals, the values recorded on the in-service test forms would show both ends have per phase voltages of nearly the exact same magnitude and phase angle. The current measurements would indicate the same magnitude but would be 180° out-of-phase. In other words, if one end read 0° (power OUT) the other end would read 180° (power IN) on their phase angle meter.

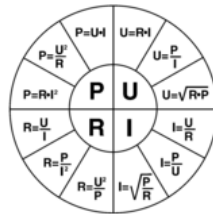
When you are taking in-service readings for substation equipment with multiple system connections, such as transformers and bus differential schemes, you take all current readings assuming current flow towards or “into” the device. It matters not that it has only two connections or several, because visualizing the assumed current direction is really no different

than previously described for in-service measurements of a transmission line connected to a substation bus. If you think about a transmission line as a single piece of power system equipment, with two system connections, you should realize that current flow was assumed into the device. You should also realize that taking in-service readings on a three terminal line would also have the assumed current flow inward from all three terminals.

Drawing, or plotting, a current vector requires special care when comparing them against voltage vectors. You draw a current vector against a voltage vector such that the current phasor points in the same direction as the voltage phasor when there is a 0° phase difference (such as for current into a purely resistive load). In visualizing an in-phase measurement (0° difference), think of a current phasor as pointing in the same direction as the voltage phasor when measuring current into and the voltage across a resistor. In this case, voltage across is also the voltage drop created by current flowing through the resistor. When loads are not purely resistive (resistance plus reactance), it is the “real” portion of the rectangular components for the AC voltage and current phasors that will be in phase. Basically, for phasor drawings of any load with an assumed direction of current flow into the load, the real component of current is drawn to point in the same direction as the real component of the voltage created by the current flow. Be aware that, when plotting phasors using rectangular coordinates, the “real” component of voltage and current will be in phase for current into a load, but will be 180° “out” if current is coming from a source.

PHASOR MATH:

In order to calculate reactance (Vars), Impedance, and more, here are some points to remember:



Formulas for Z, I, V, P, Q, VA, PF, and RF in AC:

Basic relationships

$$V=IZ,$$

$$\text{Volt-Amps}=VI$$

$$Z = \text{Impedance in ohms}$$

In addition:

$$\text{P.F.} = \text{COS } \theta$$

$$\text{R.F.} = \text{SIN } \theta$$

$$P = \text{Watts} = \text{VA} \times \text{P.F.} = VI \cdot \text{COS } \theta$$

$$Q = \text{Vars} = \text{VA} \times \text{R.F.} = VI \cdot \text{SIN } \theta$$

Apparent Power

$S = VI^*$, units VA (* means to conjugate the current angle before multiplying voltage and current vectors. Conjugate means to reverse the sign of the phase angle, change to + if -, or to - if +)

Real Power

$$P = VI \cos \theta, \text{ units Watts}$$

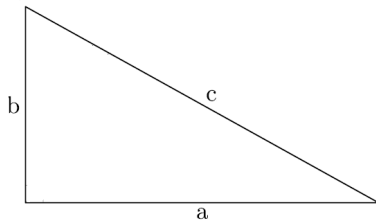
Reactive Power

$$Q = VI \sin \theta, \text{ units Var (VA reactive)}$$

Lagging load (positive vars)

Leading load (negative vars)

Pythagorean Theorem: $c^2 = a^2 + b^2$



Vectors may be expressed either in rectangular or polar coordinates.

Rectangular

$$\mathbf{R + jX}$$

$$R = Z \cos \theta$$

$$jX = jZ \sin \theta$$

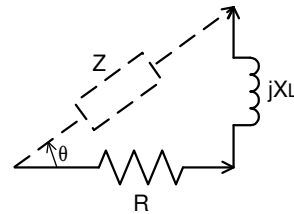
Polar

$$\mathbf{Z \angle \theta^\circ}$$

$$Z = \sqrt{R^2 + X^2}$$

$$\theta = \arctan (X/R)$$

$$\text{or } \tan^{-1} (X/R)$$



The equivalent value of the operator j is $\sqrt{-1}$. Therefore, assuming a vector of $+1 \angle 0^\circ$ or unity at zero angle, multiplying:

by j rotates it counter clockwise to a position of $1 \angle 90^\circ$,

by $j^2 = \sqrt{-1} \times \sqrt{-1} = -1$ rotates from original position to $1 \angle 180^\circ$,

and $j^3 = -1 \times j$ or 270° from the original position.

Rectangular Coordinate Math:

Adding & Subtracting (easy)

Multiplication (hard)

Division (hardest)

$$(30 + j20) + (20 + j10) = 30 + 20 + j20 + j10 = 50 + j30$$

$$(30 + j20) - (20 + j10) = 30 - 20 + j20 - j10 = 10 + j10$$

Examples of math using rectangular coordinates:

Addition:

$$\begin{array}{r} 4 + j2 \\ -2 - j3 \\ \hline 2 - j1 \end{array}$$

Subtraction:

$$\begin{array}{r} 4 + j2 \\ -2 - j3 \\ \hline 6 + j5 \end{array}$$

Multiplication:

$$\begin{array}{r} 4 + j2 \\ -2 - j3 \\ \hline -j12 - j^26 \\ -8 - j4 \\ \hline -8 - j16 - j^26 \quad (\text{since } j^2 = -1) \\ -2 - j16 \text{ is the product.} \end{array}$$

Polar Coordinate Math:

Multiplication (easy)

Division (easy)

$$10\angle 20^\circ \times 20\angle 10^\circ = (10 \times 20) \angle (20^\circ + 10^\circ) = 200\angle 30^\circ$$

$$\frac{10\angle 20^\circ}{20\angle 10^\circ} = (10 / 20) \angle (20^\circ - 10^\circ) = 0.5\angle 10^\circ$$

Examples of math converting rectangular to polar coordinates:

$$4 + j2 = A\angle\theta_1$$

$$A = \sqrt{4^2 + 2^2} = \sqrt{20} = 4.48$$

$$\theta_1 = \arctan(2/4) = \arctan 0.5 = 26.5^\circ$$

$$A = 4.48\angle 26.5^\circ$$

$$-2 - j3 = B\angle\theta_2$$

$$B = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13} = 3.61$$

$$\theta_2 = \arctan(-3/-2) = \arctan 1.5 = 56.3^\circ$$

$$B = 3.61\angle(56.3^\circ + 180^\circ)$$

Alternate example of polar to rectangular, and rectangular to polar conversion (referencing graph below):

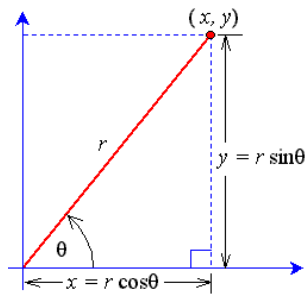
P \Rightarrow R

$$x = r \cos\theta \quad y = r \sin\theta$$

R \Rightarrow P

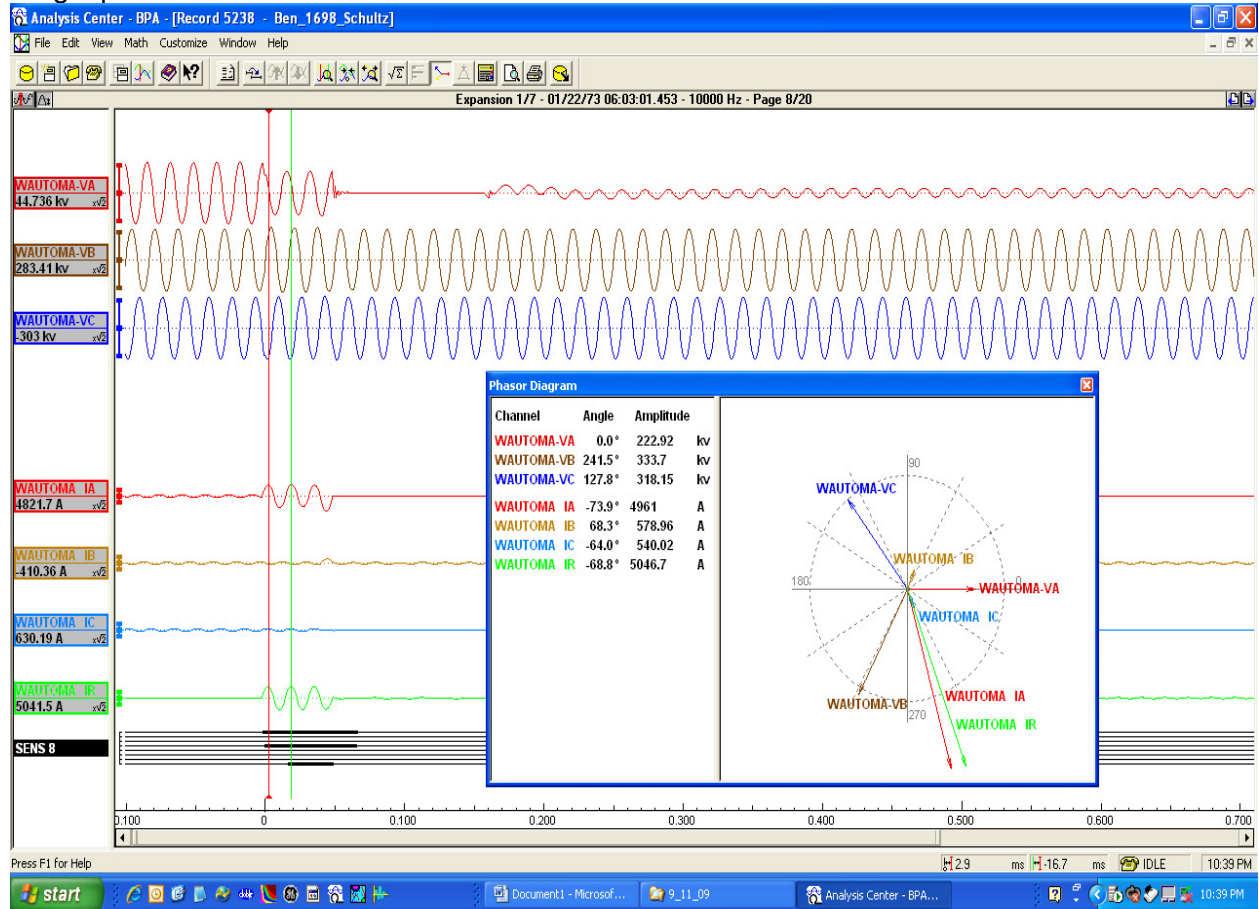
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

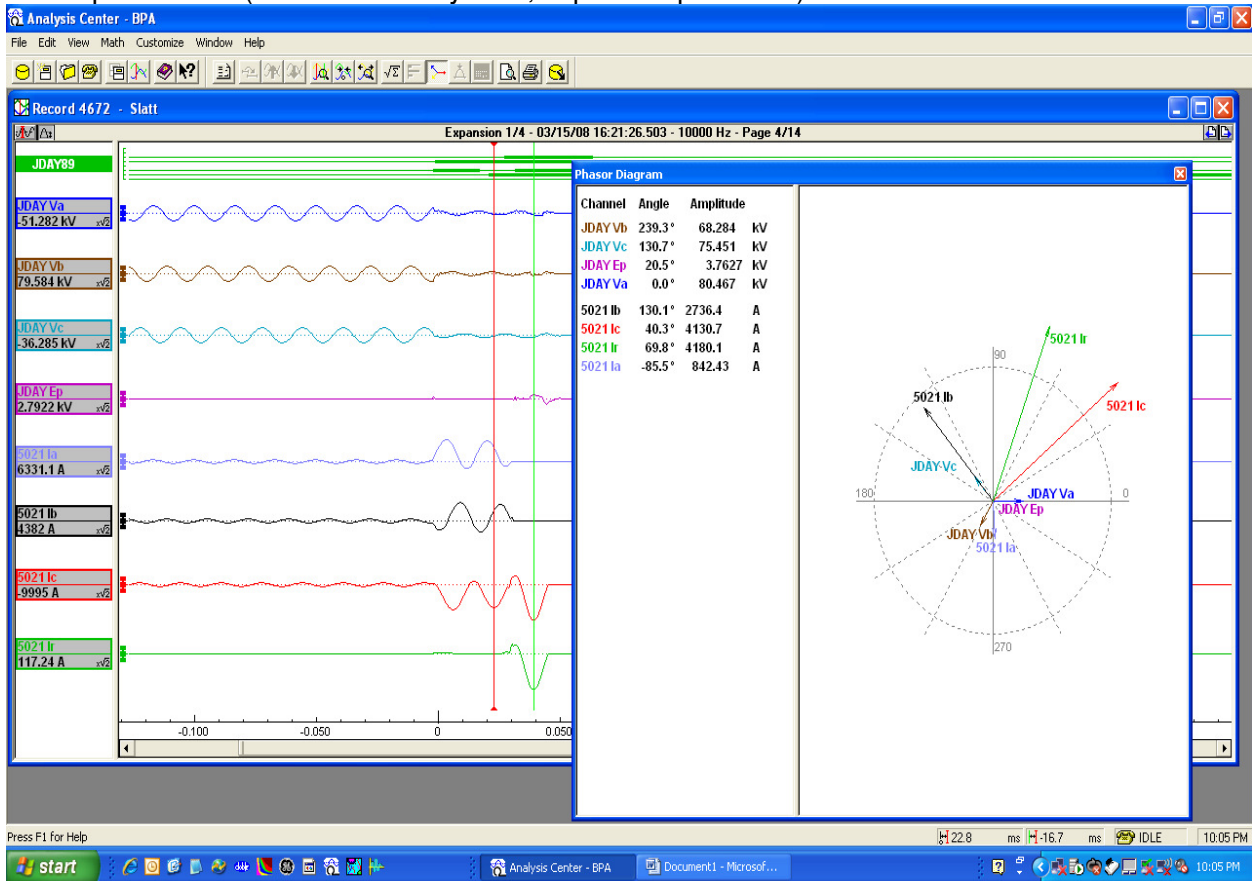


References for previous oscillography:

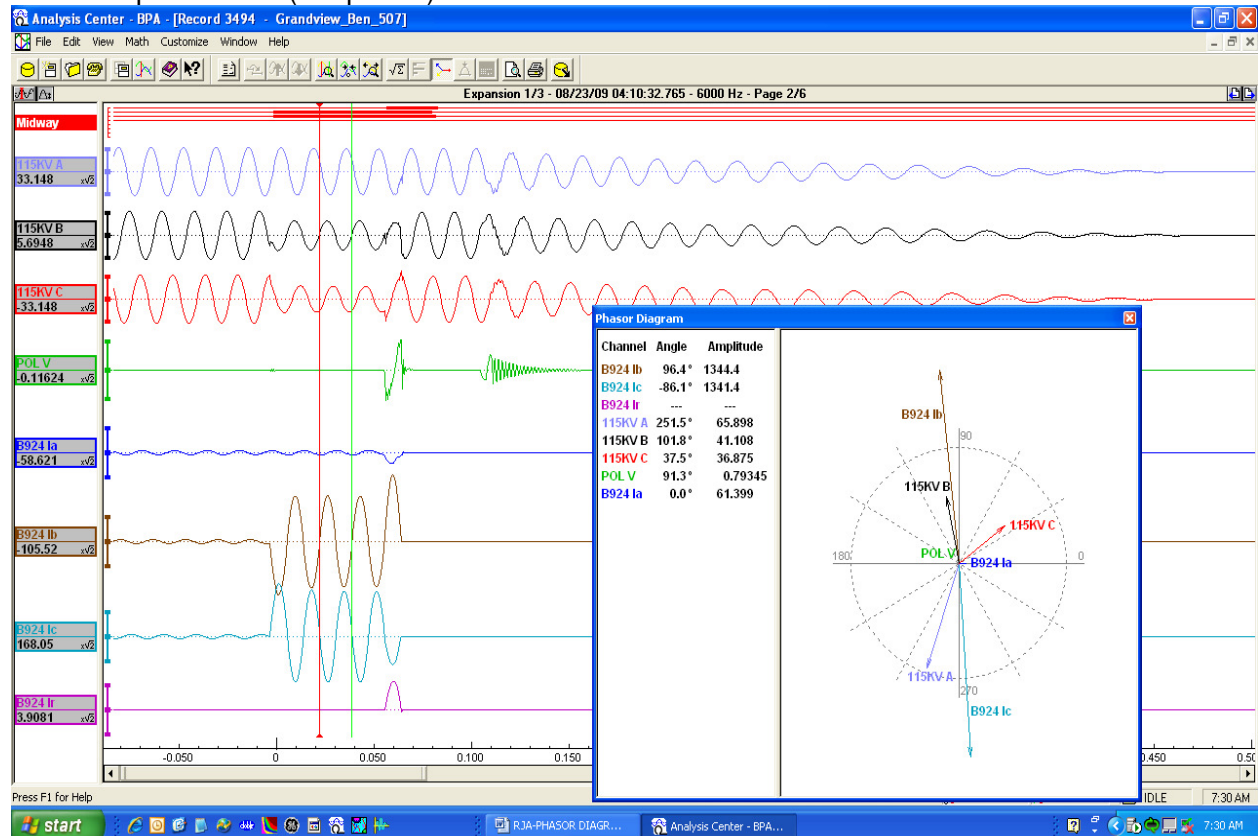
Single phase fault:



Three phase fault (on the 500kV system, C phase opens late):



Phase to phase fault (b-c phase):



Never to be overlooked are the acknowledgements and assistance by:

- Protective Relaying Principles and Applications by J. Lewis Blackburn
- Bonneville Power Administration System Protection and Control Branch
- Steve Laslo, BPA Technical Training
- Richard Becker, BPA Electrical Engineer
- Cliff Harris, Idaho Power Company